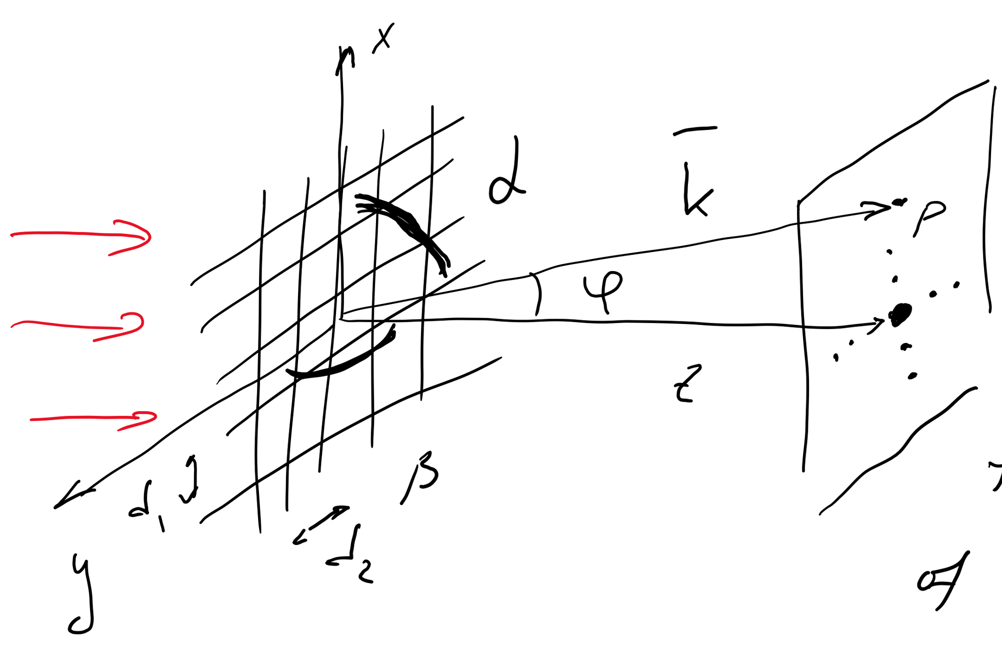
Now we will briefly discuss the case when periodic structure modulates field in whole xy plane.



For simplicity let's assume the gratings are perpendicular and there is normal incidence of light.

We will employ similar logic

$$I_p \to E_p \to E(k_x, k_y) \to E(x,y)$$

Similar to previous approach, let's define $E(x,y)$.

$$E(x,y) = E_x E_y$$

$$E_x = \begin{cases} E_0, & md_1 - \frac{b_1}{2} \le x \le md_1 + \frac{b_1}{2} \\ 0, & \text{at remaining points} \end{cases}$$

$$E_y = \begin{cases} E_0, & md_2 - \frac{b_2}{2} \le y \le md_2 + \frac{b_2}{2} \\ 0, & \text{at remaining points} \end{cases}$$

Next step is to find Fourier transformation of $E(x,y)$.

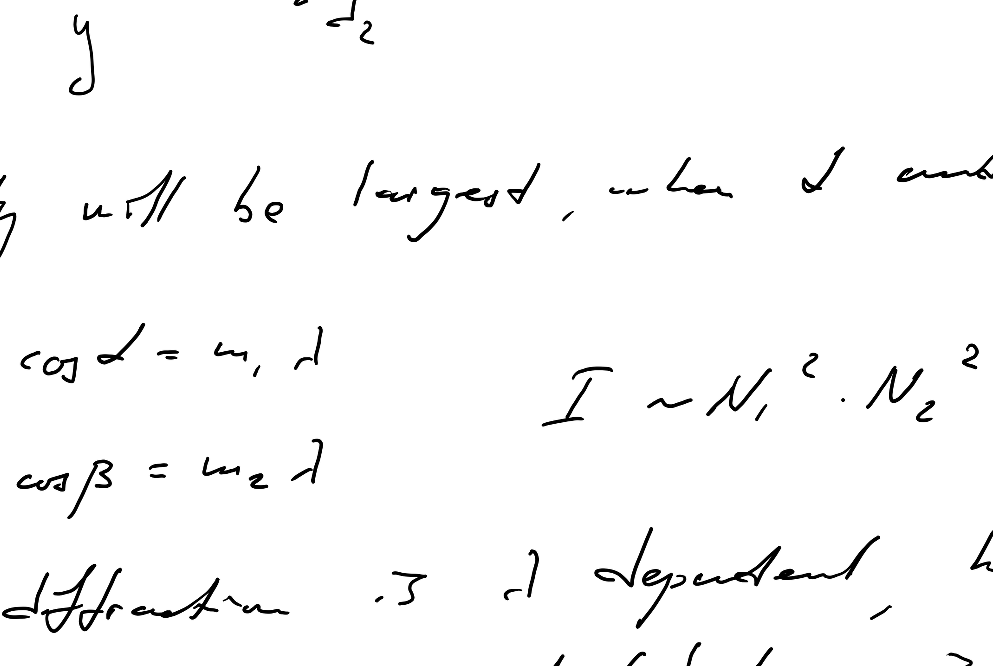
$$E(k_x, k_y) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} E(x,y) e^{-i(k_x x + k_y y)} dx dy = \int_{-\infty}^{\infty} E(x) e^{-ik_x x} dx \cdot$$

$$\cdot \int_{-\infty}^{+\infty} E(y) e^{-ik_y y} dy$$

The mathematical approach is very similar and we will write only final results.

$$I(k_x, k_y) \sim \frac{\sin^2\left(\frac{k_x N_1 d_1}{2}\right) \sin^2\left(\frac{k_y N_2 d_2}{2}\right)}{\sin^2\left(\frac{k_x d_1}{2}\right) \sin^2\left(\frac{k_y d_2}{2}\right)}, \quad \begin{cases} k_x = k\cos\alpha \\ k_y = k\cos\beta \end{cases}$$

We take away two sinc functions as they are not very important. They result in very smooth envelope function.



Intensity will be largest, when α and β are

$$\begin{cases} d_1 \cos\alpha = m_1 \lambda \\ d_2 \cos\beta = m_2 \lambda \end{cases} \qquad I \sim N_1^2 \cdot N_2^2$$

Since diffraction is λ dependent, hence we will observe spectral distribution in case of w/c.

Demonstration

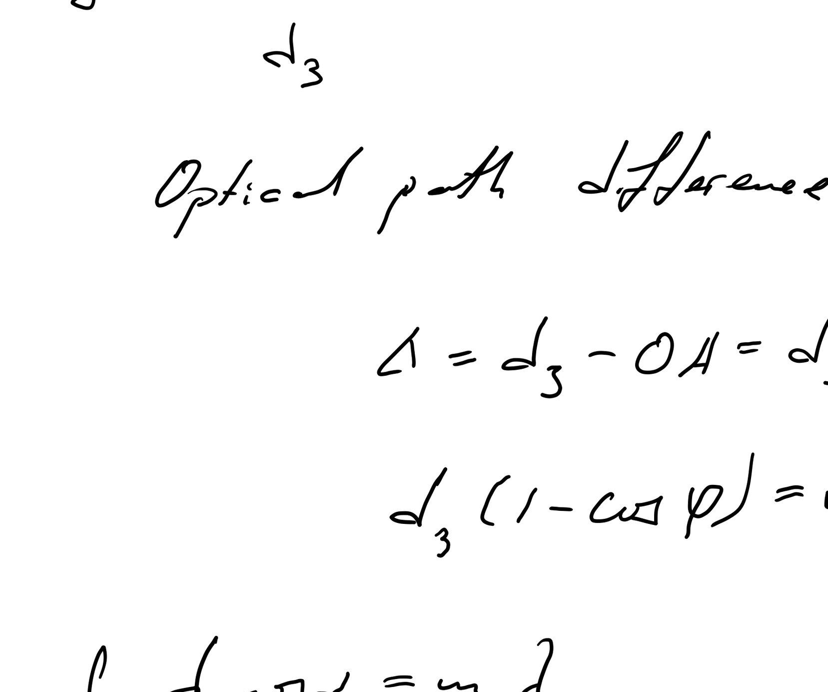
Crossed gratings.

From these experiments we can notice, that diffraction pattern depends on original structure.

$$E(x,y) = \left(\frac{1}{2\pi}\right)^2 \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} E(k_x, k_y) e^{i(k_x x + k_y y)} dk_x dk_y$$

That means we can reconstruct the object based on diffraction pattern.

In case of diffraction on volumetric periodic structure



We simply add another dimension.

Optical path difference:

$$\Delta = d_3 - OA = d_3 - d_3\cos\varphi = d_3(1-\cos\varphi)$$

$$d_3(1-\cos\varphi) = m_3\lambda$$

$$\begin{cases} d_1 \cos\alpha = m_1\lambda \\ d_2 \cos\beta = m_2\lambda \\ d_3(1-\cos\varphi) = m_3\lambda \\ \cos^2\alpha + \cos^2\beta + \cos^2\varphi = 1 \end{cases} \qquad \text{Laue equations}$$

It is important to note, that conditions for maximum are not the same for different λ.

This is different from diffraction on 1D or 2D structures.

To determine this dependence, we need to solve the above equations

$$\left(\frac{m_1\lambda}{d_1}\right)^2 + \left(\frac{m_2\lambda}{d_2}\right)^2 + \left(\frac{d_3 - m_3\lambda}{d_3}\right)^2 = 1$$

This approach allows to determine crystal structure through XRD.

Demonstration

Electron diffraction. Debye-Sherrer experiment.

Why rings change?
